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Mass sum rules for singly and doubly heavy-flavored hadrons

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Regularities in the hadron interaction energies are used to obtain formulas relating the masses of ground-state hadrons, most of which contain heavy quarks. Inputs are the constituent quark model, the Feynman-Hellmann theorem, and the structure of the colormagnetic interaction of QCD. Some of the formulas can also be obtained from heavy quark effective theory or from diquark-antiquark supersymmetry. It is argued that the sum rules are more general than the model from which they are obtained. Where data exist, the formulas agree quite well with experiment, but most of the sum rules proposed provide predictions of heavy baryon masses that will be useful for future measurements.

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I. INTRODUCTION

Quantum chromodynamics (QCD) is the generally accepted theory of strong interactions, and so, in principle, should be used to calculate hadron masses. However, thus far, nobody has succeeded in solving continuum QCD in the nonperturbative regime necessary to evaluate the masses of hadrons. This being the case, hadron masses have been calculated in lattice approximations to QCD and in various models such as potential and bag models. These approximations all have their shortcomings, the lattice calculations requiring a large amount of computer time to obtain limited results, and the model calculations being tests of the models rather than of QCD.

In the light of these difficulties, we here adopt an alternative approach of exploiting observed regularities in the properties of ground-state hadrons to predictict the masses of hadrons yet to be discovered. Our procedures should be considered complementary to other methods used for evaluating hadron masses. The observed regularities in hadrons are not in the hadron masses themselves but in interaction energies obtained by subtracting the constituent quark masses from the hadron masses.

In particular, it has been found [1,2] that if reasonable values of the quark masses are subtracted from the masses of the ground-state vector mesons, then the interaction energy is a smooth, monotonically decreasing function of the reduced mass of the constituent quarks. An analogous result has been found [2,3] for the ground-state baryons of spin 3/2, where the reduced mass is replaced by a generalized reduced mass whose inverse is the sum of the inverse quark masses. These results can be understood [2,3] from application [4] of the Feynman-Hellmann theorem [5,6] to the bound-state systems. A discussion of what constitutes a reasonable constituent quark mass is given in Ref. [2].

It is the purpose of this paper to exploit the regularities in the hadron interaction energies to obtain sum rules which relate the masses of different ground-state hadrons. The advantage in using sum rules as compared with the treatments in Refs. [1–3] is that the sum rules contain differences

of hadron masses such that the quark constituent masses cancel. Our sum rules are of three kinds: those that involve only mesons, those that involve only baryons, and those that relate the masses of baryons to the masses of mesons.

Many previous authors have obtained sum rules relating hadron masses, but, to our knowledge, most of the ones we present here are new. As an early example of a work on heavy baryon sum rules, see Franklin [7]. A few authors [8,9] have also obtained sum rules relating mesons and baryons, and Ref. [9] gives a small subset of the sum rules we obtain here. Nearly all the formulas we give involve one or more hadrons containing heavy quarks.

We motivate our sum rules from the constituent quark model, but they appear to hold because of regularities in hadron masses as a function of their valence quark content. Furthermore, some of the formulas we obtain also follow from heavy quark effective theory [10,11] or from approximate antiquark-diquark supersymmetry [12], sometimes called superflavor symmetry [13]. Therefore, we believe most of our sum rules are more general than their derivation would suggest. It must be stressed, in fact, that our results do not follow from any specific model (such as potential, lattice, flux tube etc.) but, as already mentioned, from an extended survey of the regularities of the hadron spectrum in flavor space and from general theorems. This technique may not be the most satisfactory one from the mathematical point of view (since our sum rules are not, strictly, derived from first principles), but, on the other hand, as emphasized by Martin and Richard in the context of one of our earlier predictions (obtained in similar way), our result "...not surprisingly, comes very close to the (preliminary) experimental mass" [14]. The effectiveness of our sum rules is, we believe, a sufficient motivation to propose them. They will prove extremely useful when the systematic search for heavy hadrons will have reached maturity: it will then be very convenient to have a consistent set of predictions not linked to any specific theoretical model but based rather on general properties.

As we have remarked, application of the Feynman-Hellmann theorem provides motivation for the systematic study of the vector mesons and spin-3/2 baryons. However, experimental regularities also appear in the masses of pseudoscalar mesons and spin-1/2 baryons. For this reason we also propose sum rules involving the masses of these particles. Still other sum rules concern the masses of ground-state hadrons averaged over their various spin states. For most mesons and for those baryons containing three different quark flavors, we can obtain unique expressions for the spin-averaged masses in terms of the physical hadron masses [15]. Our sum rules for spin-averaged masses are restricted to those cases in which spin averaging is possible in terms of physical masses.

In order to obtain spin averages, we assume, first, that the spin splittings of hadrons with a given quark content arise from the colormagnetic interaction of QCD; and, second, that the colormagnetic interaction can be treated perturbatively [16]. The first of these assumptions is good for the ground-state hadrons because, in first approximation, the hadron wave functions have no orbital angular momentum, and so tensor and spin-orbit forces do not contribute. Because the colormagnetic interaction between two quarks goes inversely as the product of their masses, the perturbative approximation improves as the quark masses increase. However, even though the perturbative approximation might not be good for hadrons containing only light quarks, nevertheless, the empirical regularities in hadron interaction energies persist. Because our mass formulas really rely on these regularities and not on perturbation theory, these formulas should be good even where a perturbative treatment of the colormagnetic interaction breaks down.

We can somewhat improve the perturbative expression of QCD for the colormagnetic splitting between different hadron spin states. We accomplish this by including parameters whose values are determined by fits to known data, obtaining semiempirical mass formulas for meson and baryons spin splittings [17,2,3]. Using these formulas, we can, for example, predict the masses of all hadrons of a given spin multiplet if the mass of one member is known from experiment. These results give us additional guidance in obtaining sum rules.

In Sec. 2 we discuss our methods in more detail. In Sec. 3 we give sum rules for mesons, in Sec. 4 we give sum rules for baryons, and in Sec. 5 we give sum rules relating the masses of baryons to those of mesons. In Sec. 6 we test a few of our sum rules against known data [18,19] and use other formulas to give predictions for the masses of as yet unobserved hadrons. We also discuss our

results.

II. OBTAINING HADRON MASS SUM RULES

The mass M_M of a meson and the mass M_B of a baryon presumably can be calculated in principle from QCD and can be approximately calculated in practice from lattice QCD or from a potential model or other model motivated by QCD. However, here we take a different approach, and exploit regularities in the masses of ground-state hadrons [2,3] to obtain sum rules relating their masses.

According to QCD, a meson contains a valence quark and antiquark plus a sea of gluons and quark-antiquark pairs. Similarly, a baryon contains three valence quarks plus a sea of gluons and pairs. In the constituent quark model, we neglect the sea, and take a hadron to be composed only of valence quarks with constituent masses that are a few hundred MeV larger than current quark masses. We assume that the constituent quarks can be assigned unique masses independent of the hadrons in which the quarks are bound. As we shall see, this turns out to be a very fruitful and economical assumption even though, strictly, it is probably only approximately true.

Within this framework, we introduce meson and baryon *interaction energies* $E(ij)$ and $E(ijk)$, defined by

$$E(ij) = M_M(ij) - m_i - m_j, \tag{1}$$

$$E(ijk) = M_B(ijk) - m_i - m_j - m_k, \tag{2}$$

where the symbols i, j, and k denote the valence quarks (or antiquarks) and m_i, m_j, and m_k denote their constituent masses. It has been shown [2, 3] that, for "reasonable" choices of the constituent quark masses, the interaction energies $E(ij)$ of observed vector mesons and the $E(ijk)$ of observed spin-3/2 baryons are smooth, monotonically decreasing functions of a generalized reduced mass μ:

$$dE(ij)/d\mu \leq 0, \quad dE(ijk)/d\mu \leq 0, \tag{3}$$

where μ is defined by

$$1/\mu = \sum_i 1/m_i. \tag{4}$$

The fits to the data in Refs. [2] and [3] also have the property that the interaction energies are concave upward as a function of μ:

$$d^2E(ij)/d\mu^2 \geq 0, \quad d^2E(ijk)/d\mu^2 \geq 0. \tag{5}$$

Furthermore, it was shown in Ref. [3] that the inequalities (3) and (5) still hold if the constituent quark masses are taken to be the same in mesons and baryons. The regularities in the interaction energies can be generalized to as yet undiscovered mesons and baryons.

The motivation for examining the dependence of the interaction energies as a function of μ comes from the Feynman-Hellmann theorem [5,6]. This theorem enables us to obtain the inequalities (3) in a Hamiltonian formalism with certain restrictions on the flavor-dependence of the interaction [2,3].

It was perhaps not emphasized sufficiently in Refs. [2] and [3] that *experimental* values [18,19] of the $M_M(ij)$ and $M_B(ijk)$ lead to the inequalities (3) and (5) provided the quarks are assigned constituent mass values which satisfy certain constraints obtained from the Feynman-Hellmann theorem. But the experimental hadron masses presumably are calculable in full QCD and do not depend on the constituent quark model. Therefore, the smoothness of the interaction energies $E(ij)$ and $E(ijk)$ defined in Eqs. (1) and (2) cannot really depend on the constituent quark model, even though constituent quark masses appear in these equations. Experiment is telling us simply that if reasonable values of constituent quark masses are subtracted from ground-state vector meson or spin-3/2 baryon masses, the resulting quantities are smooth, monotonically decreasing functions of μ. The use of the phrase "...reasonable values of constituent quark masses" hides a significant

subtlety. It turns out, in fact, that fits to the spectroscopic data do not well determine *absolute* values of the constituent quark masses [2]. Much more constrained from the data are the quark mass *differences*. Nevertheless, as shown in Ref. [2] one finds that some sets of quark masses found in the literature are compatible with our general theoretical considerations while other sets are not.

The principal contribution to the spin-dependent splitting of ground-state hadrons with the same quark content comes from the colormagnetic interaction of QCD [16]. In Refs. [2] and [3] it was shown that the structure of the colormagnetic interaction is such that the Feynman-Hellmann theorem does not require that that the interaction energies $E(ij)$ and $E(ijk)$ of the pseudoscalar mesons and spin-1/2 baryons be monotonically decreasing functions of μ. Nevertheless, we find empirically that even in these cases some regularities exist in the hadron interaction energies. This fact allows us to propose additional sum rules for pseudoscalar mesons and spin-1/2 baryons. Furthermore, we propose still other sum rules for hadron masses averaged over spin, some of which have appeared previously [9]. Because of the structure of the colormagnetic interaction, the spin averages of mesons and of baryons containing quarks of three different flavors can be taken with a unique result in a perturbative approximation [15].

We also make use of a result of Bertlmann and Martin [20] that in a nonrelativistic approximation, the spin-averaged meson masses satisfy the inequalities ($m_i < m_j < m_k$):

$$2M(ij) - M(ii) - M(jj) \geq 0, \tag{6}$$

$$M(ii) + M(jk) - M(ij) - M(ik) \leq 0. \tag{7}$$

Furthermore, we use a generalization of (6) and (7) to baryons [21]:

$$2M(ijk) - M(ijj) - M(ikk) \geq 0. \tag{8}$$

$$M(iii) + M(ijk) - M(iij) - M(iik) \leq 0. \tag{9}$$

Where data are available, it turns out that the inequalities (6)–(9) hold also for hadrons of definite spin configuration. We conjecture that these inequalities will hold for as yet undiscovered ground-state hadrons. Using (1) and (2), we see that (6)–(9) should hold for the corresponding meson and baryon interaction energies because the quark masses cancel in these expressions (which, among other things, greatly reduces the ambiguities that could follow from the aforementioned freedom in choosing an *absolute* scale of quark masses).

We use these ideas to motivate our proposed sum rules for hadron masses. In particular, we use quantitative estimates of the behavior of $E(ij)$, $E(ijk)$ and their first and second derivatives taken from the known hadrons and extrapolate to unknown hadrons. All the sum rules of this paper contain the masses of three or four hadrons except those involving spin-averaged masses. In order to obtain spin averaged masses, we must double the number of mesons and triple the number of baryons appearing in each sum rule [15].

Using the methods of this paper we can obtain still more complicated sum rules involving even more hadrons, but believe it is not interesting to exhibit them here.

III. MESON SUM RULES

We begin with the mesons, for which we can find only a few sum rules. From the Feynman-Hellmann theorem we deduce that inequality (6) is most likely to be an equality in the case in which the value of μ in the different terms varies least. This is the case in which the i quark is either u or d and the j quark is s. (We neglect the mass difference between u and d, and denote these quarks by q.) Using (6) in the form of an equality for vector mesons, we obtain

$$2K^*(qs) - \phi(ss) - \rho(qq) = 0, \tag{10}$$

where here and subsequently we let the symbol for a hadron denote its mass. We put the quark content in parentheses only the first time we use the symbol for a hadron, and we omit the bar on the symbol for an antiquark.

This sum rule agrees quite well with the data [18], but it does not involve mesons containing heavy quarks and may not be new. Because of the larger changes in μ for the heavy c or b quarks, sum rules analogous to Eq. (9) do not hold for mesons containing these quarks, but only the inequality (6). In obtaining (10), we assume that the ϕ is an $s\bar{s}$ state. We cannot obtain a corresponding sum rule for the pseudoscalars, because neither the η nor the η' is a pure $s\bar{s}$ state. (They are mixtures of $q\bar{q}$ and $s\bar{s}$ and may contain some admixture of glueball as well.)

A sum rule (connected with Eq. (7)) for vector mesons containing one heavy quark, is

$$D_s^*(sc) - D^*(qc) + B^*(qb) - B_s^*(sb) = 0 \tag{11.1}$$

An analogous formula for pseudoscalar mesons with the same quark content is

$$D_s - D + B - B_s = 0. \tag{11.2}$$

In our way of reasoning, these sum rules hold approximately because the values of μ for the different mesons are such that the interaction energies cancel to a good approximation. We are led to propose them by examining the systematics of the variation of the interaction energy with μ. According to heavy quark effective theory [10,11,13], the interaction energy in a hadron should not change appreciably when a c quark is replaced by a b quark. Because the quark masses cancel in (11), this sum rule follows immediately from heavy quark theory for the spin-averaged masses. We are proposing the stronger condition that analogous sum rules hold separately for the vectors and pseudoscalars. The separate sum rules follow from heavy quark effective theory if the difference in the colormagnetic energy between a meson containing a c quark and a meson containing a b quark is neglected.

All the particles appearing in the combinations (11.1,2) have by now been identified experimentally so that Eqs. (11.1,2) can be tested. The result of the comparison with the data is shown in Table I (where all the comparisons between the data, when available, and the sum rules to be discussed in what follows, are also given).

IV. BARYON SUM RULES

We have many more possibilities to obtain sum rules for baryons than for mesons because for a fairly large number of baryons, the variation of the generalized μ is small. Examining the systematics, we find that (8) is approximately an equality if none of the quarks is heavy. We obtain the following two sum rules for spin-3/2 baryons (as already noted, we put the quark content in parentheses only the first time the symbol for a hadron appears in this paper):

$$2\Sigma^*(qqs) - \Delta(qqq) - \Xi^*(qss) = 0, \tag{12}$$

$$2\Xi^* - \Sigma^* - \Omega(sss) = 0, \tag{13}$$

These two sum rules are well known, and together are just the Gell-Mann–Okubo baryon decuplet mass formula.

We next turn to sum rules involving at least two baryons containing a heavy quark. We obtain the following sum rules from the systematics:

$$\Omega + \Sigma_b^*(qqb) - \Xi^* - \Xi_b^*(qsb) = 0, \tag{14.1}$$

$$\Sigma_c^*(qqc) + \Omega_b^*(ssb) - \Xi_c^*(qsc) - \Xi_b^* = 0, \tag{14.2}$$

$$\Xi_c^* + \Xi_b^* - \Omega_c^*(ssc) - \Sigma_b^* = 0, \tag{14.3}$$

$$\Omega + \Xi_c^* - \Xi^* - \Omega_c^* = 0, \tag{14.4}$$

$$\Xi^* + \Sigma_b^* - \Sigma^* - \Xi_b^* = 0, \tag{14.5}$$

$$\Xi^* + \Xi_c^* - \Sigma^* - \Omega_c^* = 0. \tag{14.6}$$

Because each of these sum rules contains the mass of a baryon not yet discovered, they cannot be tested at this time. On the other hand, these relations (like most of those that follow) provide an approximate value for the masses of the as yet unknown baryons and will therefore await experimental verification. Our predictions are summarized in Table II.

We also have sum rules involving spin-1/2 baryons. They are somewhat different from those in Eq. (14), however, because the colormagnetic energies, which depend on quark masses and spin configuration, are different. If all three quarks in a baryon have different flavors, then two distinct baryons exist with a given quark content. In order to distinguish between these two states, it is convenient to order the quarks in a baryon such that the two lightest quarks are the first two. For example, in the case of the Λ and Σ^0, the first two quarks are u, d and the third quark is s. In the Λ, the first two quarks have spin 0, and in the Σ, the first two quarks have spin 1. Because we do not distinguish between the mass of the u and d quark, we write the quark content of the Λ and Σ as qqs. For any pair of spin-1/2 baryons containing three different flavors of quarks, the lighter baryon is the one in which the first two quarks have spin 0, and the heavier baryon is the one in which the first two quarks have spin 1, as with the Λ and Σ. We have not been able to find any sum rules involving four baryons in the case in which the first two quarks have spin 0 (Λ-type symmetry).

The sum rules for the case in which the first two quarks have spin 1 (Σ-type symmetry) are

$$\Sigma(qqs) + \Omega_c(ssc) - \Xi(ssq) - \Xi_c'(qsc) = 0, \tag{15.1}$$

$$\Xi + \Sigma_c(qqc) - \Sigma - \Xi_c' = 0, \tag{15.2}$$

$$\Xi + \Xi_b'(qsb) - \Sigma - \Omega_b(ssb) = 0, \tag{15.3}$$

$$\Xi_c' + \Xi_b' - \Sigma_b(qqb) - \Omega_c = 0, \tag{15.4}$$

$$\Omega_c + \Sigma_b - \Sigma_c - \Omega_b = 0. \tag{15.5}$$

(Here and in the following, when two spin-1/2 baryons exist with the same quark content and the same Greek symbol [18], a prime denotes the configuration in which the first two quarks have spin 1.)

Only the last of the sum rules in (15) follows from heavy quark theory, and then only in the approximation of neglecting certain differences in colormagnetic energy. Data do not yet exist to test these sum rules. We expect that all of the mass formulas in (14) and (15) will hold to about 20 MeV or better. Again, when applicable, our predictions are listed in Table II.

We have been able to find a rather large number of sum rules involving some baryons which contain two heavy quarks. Although several theoretical papers have been written about baryons containing two heavy quarks [7,8,13,22,23], none has yet been observed. We list our sum rules for these baryons in an Appendix.

V. SUM RULES FOR BARYONS AND MESONS

We have obtained an even larger number of mass formulas involving two baryons and two mesons than the number involving four baryons. We give here those in which no baryon contains more than one heavy quark, and relegate to the Appendix the formulas involving baryons containing two heavy quarks.

The formulas are of four kinds: 1) those with spin-3/2 baryons and vector mesons, 2) those with spin-1/2 baryons (Σ-type symmetry) and pseudoscalar mesons, 3) those with spin-1/2 baryons (Λ-type symmetry) and pseudoscalar mesons, and 4) those involving spin-averaged baryons and mesons.

We begin with formulas for spin-3/2 baryons and spin-1 mesons. Our sum rules are

$$\Xi^* - \Xi_c^* + D^*(qc) - K^* = 0, \tag{16.1}$$

$$\Omega - \Omega_b^* + B^*(qb) - K^* = 0, \tag{16.2}$$

$$\Sigma_c^* - \Xi^* + \phi - D^* = 0, \tag{16.3}$$

$$\Sigma_c^* - \Xi_c^* + K^* - \rho = 0, \tag{16.4}$$

$$\Sigma_c^* - \Sigma_b^* + B^* - D^* = 0, \tag{16.5}$$

$$\Omega_c^* - \Xi^* + \rho - D^* = 0, \tag{16.6}$$

$$\Omega_c^* - \Xi_c^* + K^* - \phi = 0, \tag{16.7}$$

$$\Omega_c^* - \Xi_b^* + B^* - D_s^*(sc) = 0, \tag{16.8}$$

$$\Sigma_b^* - \Sigma_c^* + D_s^* - B_s^*(sb) = 0, \tag{16.9}$$

$$\Xi_b^* - \Omega + \phi - B^* = 0, \tag{16.10}$$

$$\Xi_b^* - \Omega_b^* + K^* - \rho = 0, \tag{16.11}$$

$$\Omega_b^* - \Omega_c^* + D^* - B^* = 0, \tag{16.12}$$

$$\Omega_b^* - \Sigma_b^* + \rho - \phi = 0. \tag{16.13}$$

Of these sum rules, (16.5), (16.8), (16.9), (16.10), and (16.12) can be justified by antiquark-diquark supersymmetry. If the contribution from heavy quarks to the colormagnetic energy is neglected, then (16.5), (16.9), and (16.12) also follow from heavy quark effective theory. The masses of all hadrons appearing in the sum rules (16.3,5,9) are known from experiment, and these sum rules are well satisfied (as shown in Table I).

The formulas involving two spin-1/2 baryons (Σ-type symmetry) and two pseudoscalar mesons are:

$$\Xi_c' - \Omega_b + B_s - D = 0, \tag{17.1}$$

$$\Sigma_b - N(qqq) + K - B_s = 0, \tag{17.2}$$

$$\Xi_b' - \Sigma_c + D - B_s = 0. \tag{17.3}$$

Of these, the first and the third descend from antiquark-diquark supersymmetry.

We were unable to find formulas involving two spin-1/2 baryons (Λ-type symmetry) containing only up to one heavy quark, and two pseudoscalar mesons. The formulas involving baryons containing two heavy quarks are reported in the appendix.

Finally, the sum rules involving spin-averaged baryons and mesons are:

$$(2\Sigma^* + \Sigma + \Lambda(qqs))/4 - (N + \Delta)/2 + (3\rho + \pi)/4 - (3K^* + K(qs))/4 = 0, \tag{18.1}$$

$$(2\Sigma_c^* + \Sigma_c + \Lambda_c)/4 - (2\Sigma^* + \Sigma + \Lambda)/4 + (3K^* + K)/4 - (3D^* + D)/4 = 0, \tag{18.2}$$

$$(2\Sigma_c^* + \Sigma_c + \Lambda_c)/4 - (2\Sigma_b^* + \Sigma_b + \Lambda_b(qqb))/4 + (3B^* + B)/4 - (3D^* + D)/4 = 0, \tag{18.3}$$

$$(2\Sigma_b^* + \Sigma_b + \Lambda_b)/4 - (2\Sigma_c^* + \Sigma_c + \Lambda_c)/4 + (3D_s^* + D_s/4) - (3B_s^* + B_s)/4 = 0. \tag{18.4}$$

The first three of these equations have been obtained previously [9]. The last two can also be derived from heavy quark theory. Furthermore, Eq. (18.4) follows from (18.3) with the help of (11.1) and (11.2). We do, however, have several new sum rules involving spin averages of baryon and meson masses. These are given in the Appendix in Eq. (A.6).

Those sum rules for which data exist compare quite well with experiment, as can be seen from Table I. We expect that the sum rules of this section will be good to 20 MeV or better.

VI. RESULTS AND DISCUSSION

We have not verified that all the sum rules of this paper are linearly independent, but we believe that most, if not all, of them are, with the exception of the formula given in Eq. (18.4), which we have explicitly pointed out is derivable from others. If a few sum rules should turn out to be linear combinations of others, no harm is done.

Some of the formulas given in the preceding three sections contain only masses of known hadrons. We test these formulas using data from the Particle Data group [18] and more recent preliminary data from a conference talk by Jarry [19]. We give our results in Table I. As can be seen from this table, those of our sum rules which can be tested agree with experiment within about 10 MeV or better.

If all but one of the hadrons entering a formula have been observed, we use the formula to predict the mass of the unknown hadron. We go on to use these predicted masses in other sum rules to obtain still further predictions. In following this method, we use the sum rules of the appendix as well as those appearing in the main body of the paper. The predicted masses arising from this procedure are given in Table II. Our estimated errors are 20 MeV or less for sum rules not involving any baryons containing two heavy quarks. The caption to Table II gives our estimated errors in all cases.

It is evident from Table II that we are unable to use our mass formulas to get predictions for the baryons Ξ_{bb} and Ω_{bb}. However, we can estimate the difference $\Omega_{bb} - \Xi_{bb}$. From heavy quark symmetry, it should be equal to $\Omega'_{cb} - \Xi'_{cb} = 95$ MeV, while antiquark-diquark supersymmetry suggests that $\Omega_{bb} - \Xi_{bb} = B_s - B = 90$ MeV.

We may consider the question: If any of our sum rules should turn out to be badly in error, would we learn anything? First, we believe it is highly unlikely that such an event will happen, because enough hadrons are already known to give us confidence that the regularities in the interaction energies are much more than coincidence. Therefore, these regularities should persist in ground-state hadrons not yet discovered. Second, in the unlikely event that an exception is found, it will cast doubt on the flavor-independence of the fundamental interaction between quarks.

In conclusion, relying on observed systematics of the interaction energies of known hadrons, we have obtained a large number of sum rules for the masses of known and unknown hadrons. In those cases in which the masses of all hadrons entering our sum rules are known from experiment, the sum rules agree with experiment to about 10 MeV or better. This fact gives us confidence that the predictions of unknown hadron masses which follow from the sum rules are likely to be correct within quite small errors compared to the masses themselves. We believe our predictions should be a useful guide to experimentalists searching for new hadrons, as our results depend on the regularities in observed hadrons persisting to hadrons not yet seen rather than on any specific model of quark interactions.

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APPENDIX

In this Appendix we give sum rules involving at least two baryons containing two heavy quarks. We expect that the sum rules involving baryons containing no b quarks are good to 30 MeV, those

involving baryons containing no more than one b quark are good to about 40 MeV, and those involving baryons containing two b quarks are good to about 50 MeV.

The formulas for spin-3/2 baryons are:

$$\begin{aligned} \Omega + \Xi^*_{cc}(ccq) - \Xi^*_c - \Omega^*_c &= 0, \\ \Omega + \Xi^*_{cb}(qcb) - \Xi^*_c - \Omega^*_b &= 0, \\ \Sigma^*_c + \Xi^*_{cb} - \Sigma^*_b - \Xi^*_{cc} &= 0, \\ \Sigma^*_c + \Omega^*_{cb}(scb) - \Sigma^*_b - \Omega^*_{cc}(ccs) &= 0, \\ \Sigma^*_c + \Xi^*_{bb}(bbq) - \Sigma^*_b - \Xi^*_{cb} &= 0, \\ 2\Xi^*_c - \Xi^* - \Xi^*_{cc} &= 0, \\ \Omega^*_c + \Sigma^*_b - \Xi^* - \Xi^*_{cb} &= 0, \\ \Omega^*_c + \Xi^*_b - \Omega - \Xi^*_{cb} &= 0, \\ \Sigma^*_b + \Omega^*_{cb} - \Sigma^*_c - \Omega^*_{bb}(bbs) &= 0, \\ \Omega^*_b + \Xi^*_{cb} - \Omega^*_c - \Xi^*_{bb} &= 0. \end{aligned} \tag{A.1}$$

The formulas for spin-1/2 baryons (Σ-type symmetry) are:

$$\begin{aligned} \Xi'_c + \Xi'_{cb}(qcb) - \Xi'_b - \Xi_{cc}(ccq) &= 0, \\ \Sigma_b + \Omega_{cc}(ccs) - \Sigma_c - \Omega'_{cb}(scb) &= 0, \\ \Omega_b + \Xi_{cc} - \Xi'_b - \Omega_{cc} &= 0, \\ \Omega_b + \Omega_{cc} - \Omega_c - \Omega'_{cb} &= 0. \end{aligned} \tag{A.2}$$

We now turn to the mass sum rules involving two baryons and two mesons. For spin-3/2 baryons and vector mesons, we have:

$$\begin{aligned} \Sigma^*_c - \Xi^*_{cb} + B^* - \rho &= 0, \\ \Xi^*_c - \Xi^*_{cc} + D^* - K^* &= 0, \\ \Xi^*_c - \Omega^*_{cc} + D^*_s - K^* &= 0, \\ \Xi^*_c - \Xi^*_{cb} + B^* - K^* &= 0, \\ \Xi^*_c - \Omega^*_{cb} + B^*_s - K^* &= 0, \\ \Omega^*_c - \Xi^*_{cb} + B^* - \phi &= 0, \\ \Omega^*_c - \Omega^*_{cb} + B^*_s - \phi &= 0, \\ \Sigma^*_b - \Xi^*_{cb} + D^* - \rho &= 0, \\ \Sigma^*_b - \Xi^*_{bb} + B^* - \rho &= 0, \\ \Xi^*_{cc} - \Sigma^*_c + K^* - D^*_s &= 0, \\ \Omega^*_{cc} - \Xi^*_{cc} + B^* - B^*_s &= 0, \\ \Omega^*_{cc} - \Omega^*_{cb} + B^* - D^* &= 0, \\ \Xi^*_{cb} - \Xi^*_b + \phi - D^*_s &= 0, \\ \Xi^*_{cb} - \Omega^*_{bb} + B^*_s - D^* &= 0, \\ \Omega^*_{cb} - \Omega^*_{bb} + B^* - D^* &= 0, \\ \Omega^*_{bb} - \Omega^*_{cb} + D^*_s - B^*_s &= 0. \end{aligned} \tag{A.3}$$

The sum rules involving spin-1/2, Σ-type symmetry baryons and pseudoscalar mesons are:

$$\begin{aligned} N - \Xi'_{cb} + B_c - \pi &= 0, \\ \Sigma - \Xi_{cc} + \eta_c(cc) - K &= 0, \\ \Omega_{cc} - \Xi_{cc} + D - D_s &= 0, \\ \Xi'_{cb} - \Omega'_{cb} + B_s - B &= 0, \\ \Xi_{bb}(bbq) - \Omega_{bb}(bbs) + B_s - B &= 0. \end{aligned} \tag{A.4}$$

The sum rules involving spin-1/2, Λ-type symmetry baryons and pseudoscalar mesons are:

$$\begin{aligned} \Xi_{cb}(qcb) - \Omega_{cb}(scb) + D_s - D = 0, \\ \Omega_{cb} - \Xi_{cb} + B - B_s = 0. \end{aligned} \tag{A.5}$$

The formulas involving spin averages are:

$$\begin{aligned} (2\Sigma_c^* + \Sigma_c + \Lambda_c)/4 - (2\Xi_{cb}^* + \Xi_{cb}' + \Xi_{cb})/4 + (3B_s^* + B_s)/4 - (3K^* + K)/4 = 0, \\ (2\Sigma_b^* + \Sigma_b + \Lambda_b)/4 - (2\Xi_{cb}^* + \Xi_{cb}' + \Xi_{cb})/4 + (3D_s^* + D_s)/4 - (3K^* + K)/4 = 0, \\ (2\Xi_{cb}^* + \Xi_{cb}' + \Xi_{cb})/4 - (2\Omega_{cb}^* + \Omega_{cb}' + \Omega_{cb})/4 + (3D_s^* + D_s)/4 - (3D^* + D)/4 = 0, \\ (2\Xi_{cb}^* + \Xi_{cb}' + \Xi_{cb})/4 - (2\Omega_{cb}^* + \Omega_{cb}' + \Omega_{cb})/4 + (3B_s^* + B_s)/4 - (3B^* + B)/4 = 0. \end{aligned} \tag{A.6}$$

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TABLE I. Test of sum rules when all particles' masses are known from experiment. Column 1 refers to the equation number; column 2 lists the actual value in MeV of the violation of the mass formula and its error, obtained by using experimental mass values from the Particle Data Group [18] and from a recent conference report by Jarry [19].

Eq. #	Violation in MeV
(10)	1 ± 3
(11.1)	5 ± 5 or $12 \pm 4^*$
(11.2)	4 ± 5 or $11 \pm 2^*$
(12)	5 ± 3
(13)	9 ± 3
(16.3)	8 ± 2
(16.5)	$4 \pm 18^*$
(16.9)	$8 \pm 18^*$
(17.2)	$6 \pm 18^*$
(18.1)	1 ± 3
(18.2)	2 ± 3
(18.3)	$8 \pm 18^*$
(18.4)	$3 \pm 18^*$

*Results which use any data from Ref. [19] are marked with an asterisk. The sum rules in Eqs. (16.5), (16.9), (17.2), (18.3), and (18.4) cannot be tested solely with the data from Ref. [18].

TABLE II. Predicted baryon masses in MeV. Here M_A and M_S denote the two spin-1/2 baryons (with Λ- and Σ-type symmetry respectively), M^* denotes spin-3/2 baryons, and M_B denotes baryon masses spin-averaged according to the presciption of Ref. [15]. The predicted mass values are determined, wherever possible, using mass sum rules in which the values of all masses but one are known from experiment. If more than one such formula exists for a given hadron, an average of the various predictions is taken. The results obtained this way are exploited to obtain new mass values from sum rules containing more than one hadron with mass not yet measured. The errors in the predicted masses are estimated to be on the order of 20 MeV or less for a baryon containing up to one heavy quark; on the order of 30 MeV if the baryon contains two c quarks, 40 MeV if one c and one b quarks are present, and 50 MeV if there are two b quarks.

Quark content and symbol	M_A	M_S	M^*	M_B	M_A	M_S	M^*
qqq		N	Δ	1086		939^a	1232^a
qqs	Λ	Σ	Σ^*	1270	1116^a	1193^a	1385^a
ssq		Ξ	Ξ^*			1318^a	1533^a
sss			Ω				1672^a
qqc	Λ_c	Σ_c	Σ_c^*	2450	2285^a	2453^a	2530^a
qsc	Ξ_c	Ξ_c'	Ξ_c^*	2588	2468^a	2582^b	2651^b
ssc		Ω_c	Ω_c^*			2710^a	2775
qqb	Λ_b	Σ_b	Σ_b^*	5783	5627^a	5818^a	5843^a
qsb	Ξ_b	Ξ_b'	Ξ_b^*			5955	5984
ssb		Ω_b	Ω_b^*			6075	6098
ccq		Ξ_{cc}	Ξ_{cc}^*			3676	3746
ccs		Ω_{cc}	Ω_{cc}^*			3787	3851
qcb	Ξ_{cb}	Ξ_{cb}'	Ξ_{cb}^*	7062	7029	7053	7083
scb	Ω_{cb}	Ω_{cb}'	Ω_{cb}^*	7151	7126	7148	7165
bbq		Ξ_{bb}	Ξ_{bb}^*				10398
bbs		Ω_{bb}	Ω_{bb}^*				10483

^{a} Input mass from experiment [18,19].

^{b} After this work was completed, we learned that the Ξ_c' and Ξ_c^* have been observed [24]. Preliminary values of their masses are 2573 ± 4 and 2643 ± 4 respectively, in agreement with our predictions within our stated errors.